Modeling COVID-19 Transmission Dynamics with Behavioral Change

Tsz-Lik CHAN
(with Hsiang-Yu YUAN and Wing-Cheong LO)

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About compartmental models I

Generally, in compartmental models the whole population is divided into different compartments. The population is homogeneous in terms of its characteristic of interest in each compartment. For example, SIR model divides the population into susceptible, infected and recovered. For different diseases, we can add more compartments to fit a specific characteristic of the disease. Some common compartments are,

- Exposed compartment: In this compartment, the population is exposed to the disease, i.e. being infected. But are not infectious yet. For many diseases, the individual will not be immediately infectious right after being infected. The period between the individual is infected and the individual is infectious is called the latent period.

About compartmental models II

- Asymptomatic compartment: In this compartment, the population is infectious but shows no symptoms. The time between the individual is infected and the individual shows symptoms is called the incubation period. For a disease with a latent period shorter than its incubation period, that is the individual is infectious before the individual shows symptoms. It would be better to add an asymptomatic compartment.
- Vaccinated compartment: In this compartment, the population is vaccinated. And thus has a different death rate and infection rate than those who are not vaccinated.

Backgrounds about COVID-19 I

- COVID-19 is the pandemic affecting many countries.
- One characteristics of COVID-19 is the presence of asymptomatic patients. The estimation of the percentage of asymptomatic patients varies a lot, from 14% (Li et al., 2020) to as large as 90% (Gaeta et al., 2020). Asymptomatic patients have no symptoms and live like normal people. Thus the pandemic would transmit faster when there are a large number of asymptomatic patients.
- For COVID-19, most studies suggest that the latent period is around one to two days shorter than the incubation period. So that an infectious individual has around one to two days to live normally, and during this period the individual could pass the disease unconsciously.

Backgrounds about COVID-19 II

- A common disease control strategy that is implemented worldwide is social distancing. The activity of the population could affect the dynamics of the pandemics. When people have more contact with others the disease transmits faster. This is important when the population of asymptomatic patients is large. Since you are not aware whether you have contact with an infectious person or not, and there would be a high chance of contact with an asymptomatic patient.

Backgrounds about COVID-19 III

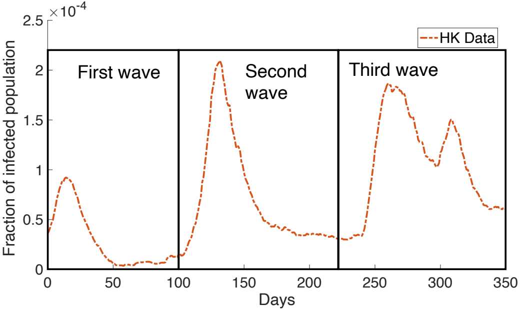

Figure: Data from the Department of Health. The infected population can be classified into three waves.

Backgrounds about COVID-19 IV

As shown in the figure, the dynamics of the infected population are recurrent. We can observe waves of the infected population. These dynamics are generally not seen in a simple SIR model. We want to capture the recurrent outbreaks by considering two aspects:

1. behavioural change,
2. asymptomatic patients.

About mean-filed game and evolution I

- We want to adapt the theory of games into the pandemic modeling. In particular, from mean-field or population games, a theory that describes a how a rational agent makes decision in a very large population.
- The model should be a system that describes how the population density evolved with time and an optimization problem that describes how an agent should maximize his payoff.
- We consider two types of strategy, either live normally or reduce their activity in response to the pandemic.
- Another important idea from evolution is that we need to allow the population to "learn", that is to change their current strategy after rational thinking. Rational thinking means the population will compare different strategies by comparing the corresponding payoffs.

About mean-filed game and evolution II

- The frequency of a particular strategy depends on the payoff. Also, the payoff of the strategy depends on the frequency of all strategies.
- Final two assumptions in our model,
  - There are only two strategies. New strategy will not emerge.
  - Agents are myopic, i.e., their discount factor is small, $\delta \to 0$

An existing model which consider behavioral change I

A recent model by Amaral et al. (2021) that borrowed ideas from evolution and game theory is:

$$
\begin{cases}
\dot{S}^n = -\beta_N I^n S^n - \beta_a I^r S^n + \tau\Phi_S \\
\dot{S}^r = -\beta_N I^n S^r - \beta_a I^r S^r + \tau\Phi_S \\
\dot{I}^n = \beta_N I^n S^n + \beta_a I^r S^r + \tau\Phi_I \\
\dot{I}^r = \beta_N I^n S^r - \beta_a I^r S^r + \tau\Phi_I \\
\dot{R} = \gamma(I^n + I^r)
\end{cases}
$$

In this model, both the susceptible and infected populations are divided into two groups.

An existing model which consider behavioral change II

- This model differs from a simple SIR model in terms of the fact that susceptible population (infected population) could move between S^n and S^r (I^n and I^r) through $\tau\Phi_S$ ($\tau\Phi_I$). And the infection rate is different in the two groups.
- The susceptible could protect themselves from infection by moving to S^r.
- Population can learn (to change their strategy from n to r or from r to n) according to the Fermi rule, $\theta(p_1, p_2)$, which gives the probability of changing from strategy with payoff p_1 to strategy with payoff p_2

$$\theta(p_1, p_2) = \frac{1}{1 + e^{-(p_2 - p_1)/k}}$$

An existing model which consider behavioral change III

- This model considers the case of social learning (i.e. learn from another individual when they meet) in which this rate of change also depends on the probability that normal type meets with reduced type

$$\Phi_S = S^r(S^n + I^n)\theta(p_r, p_n) - S^n(S^r + I^r)\theta(p_n, p_r),$$
$$\Phi_I = I^r(S^n + I^n)\theta(p_r, p_n) - I^n(S^r + I^r)\theta(p_n, p_r).$$

- This assumption is not valid in our model. We want to study a case where the individuals have the required information and the required knowledge to act rationally by themselves.
- Our model correspond to the concept of self-learning. People obtain information from e.g. newspaper or TV. And make judgement themselves using that information.

Details of the proposed model I

We consider the stage of an infected susceptible as follows,

$$\boxed{\text{Susceptible}} \rightarrow \boxed{\text{Exposed}} \rightarrow \begin{cases} \boxed{\text{Asymptomatic}} \\ \boxed{\text{Infected}} \end{cases}$$

So that after being infected, the person will become exposed (infected but not infectious). After a certain time, person will either develop symptoms and become infectious, or the person will not develop symptoms but still infectious, in this case, he will be called asymptomatic.

Details of the proposed model II

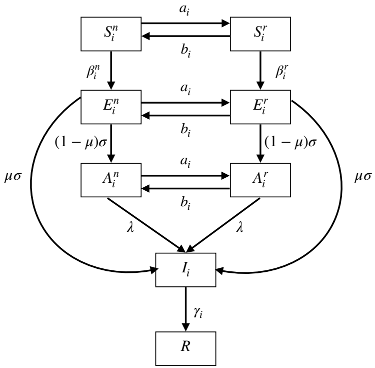

Figure: A visualization of the model.

- S, A, E, I, R are the susceptible, asymptomatic, exposed, infected, recovered respectively.
- a_i and b_i are the rate that population changes their strategy. (Rate of behavioural change.)

Details of the proposed model III

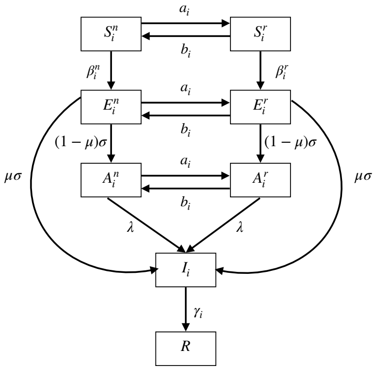

Figure: A visualization of the model.

- λ is the report rate, γ_i is the recovery rate. β_i^r, β_i^n and β_i^l are the transmission rate.
- μ is the probability of showing symptoms after infection.
- σ is the rate at which exposed population become infectious.
- Different i represents different subgroup.

Details of the proposed model IV

The proposed model

$$\begin{cases} \dot{S}_i^r = a_i S_i^n - b_i S_i^r - \beta_i^r S_i^r, \\ \dot{S}_i^n = -a_i S_i^n + b_i S_i^r - \beta_i^n S_i^n, \\ \dot{E}_i^r = a_i E_i^n - b_i E_i^r - (1-\mu)\sigma E_i^r - \mu\sigma E_i^r + \beta_i^r S_i^r, \\ \dot{E}_i^n = -a_i E_i^n + b_i E_i^r - (1-\mu)\sigma E_i^n - \mu\sigma E_i^n + \beta_i^n S_i^n, \\ \dot{A}_i^r = a_i A_i^n - b_i A_i^r + (1-\mu)\sigma E_i^r - \lambda A_i^r, \\ \dot{A}_i^n = -a_i A_i^n + b_i A_i^r + (1-\mu)\sigma E_i^n - \lambda A_i^n, \\ \dot{I}_i = \lambda(A_i^r + A_i^n) + \mu\sigma E_i^r + \mu\sigma E_i^n - \gamma_i I_i, \\ \dot{R} = \sum_{i=1}^M \gamma_i I_i. \end{cases}$$

Formulating the proposed model I

- In this section, we provide a formulation of the value function.
- The population, based on information, decide rationally by comparing payoff.
- We assume that the population are smart, so they have the required knowledge to maximize their expected payoff and minimize the expected cost. This could be model as an optimization problem.
- But the information is not prefect. There is considerably delay between when something happened and when you know something happened. As we will soon see delay in information is a critical aspect that affects the recurrent outbreak. And the population cannot get all the information they need.

Formulating the proposed model II

We define the payoff functions below.

Payoff functions

$$u_i^n(t) := \max_{z^n,\ 0 \le z^n \le 1} v_i(z, \boldsymbol{e}(t)),$$
$$u_i^r(t) := \max_{z^r,\ 0 \le z^r \le r_{max} < 1} v_i(z, \boldsymbol{e}(t))$$

- $\boldsymbol{e} = [I_1(t-\tau), \cdots, I_M(t-\tau)]$ represent the information of numbers of infected population. This information is delayed by τ time units. No information of asymptomatic population.
- By the Fermi Rule defined before, the rate of changing strategy will be

$$a_i = \theta(u_i^n(t), u_i^r(t)),$$
$$b_i = \theta(u_i^r(t), u_i^n(t)).$$

Formulating the proposed model III

- This optimization problem is solved at each time t to determine the optimal parameters z^{n*} and z^{r*}, which we can interpret it as the frequency of optimal activity for the normal type and the reduced type.
- The feasible sets for the normal type and the reduced type are different. $0 \leq z^n \leq 1$. But $0 \leq z^r \leq r_{max} < 1$.
- This means that although the reduced type can sometimes obtain more payoff by increasing the z^r above r_{max}, they are reluctant to do so. We can think of that they are conservative in making decision (risk-averse).

Formulating the value function I

Here, we introduce what are the payoffs based on, that is the value function v in the definition of u_i^n and u_i^r. The v we used is modify from Garibaldi et al. (2020) and Amaral et al. (2021).

- The idea is to measure the value that you can obtain now and in the future.
- That is the value function should be the sum of present value and the expected cost.
- Present value means the value that the population can gain immediately. e.g. The joy for having decent meal outside with your family.
- The expected cost is further decomposed into the expected cost of being susceptible, and the expected cost of being infected.
- In such case, this value is inherently abstract. It only gives preference for making decision.

Formulating the value function II

Value function

$$v_i(z) = \underbrace{\bar{v}}_{\text{Present value}} - \underbrace{(cp + (1-p))}_{\text{Future cost}},$$

$$\textit{where } c \in \mathbb{R},\ \ p = \omega m(z) \sum_{j=1}^{M} k_I I_j(t-\tau).$$

- $m(z)$ here is the average number of contact induced by z.
- p is a probability of being infected.
- c is the cost after infection, the cost of being susceptible is, relatively, 1.
- The information of I_j is delayed by time τ. It represents the information lag in reality.

Formulating the value function III

- The value represents, at z, the current reward (present value) and all the future cost. Since $u = \max_z v$, the payoff is the optimal value that the agent can obtained.

Formulating the transmission rate

The transmission rate in the proposed model are β_i^r, β_i^n. They again depend on m and the probability of contacting with infectious populations.

$$\beta_i^r(z^{r*}, t) = m(z^{r*}) \sum_{j=1}^{M} k_n A_j^n + k_r A_j^r + k_I I_j,$$

$$\beta_i^n(z^{n*}, t) = m(z^{n*}) \sum_{j=1}^{M} k_n A_j^n + k_r A_j^r + k_I I_j.$$

k_n, k_r and k_I represent the infection rate.

Parameters estimation

- In this section we will present the parameter estimation and numerical simulations with Hong Kong's data. Most parameters are adopted from various studies or approximated from data. First, we use the same form of $m(z) = 2.2z$, $\bar{v}(z) = z - \frac{z^2}{2}$ as used by Garibaldi et al. (2020).
- We take $\lambda = \frac{1}{5.7 \text{ days}}$, $\gamma = \frac{1}{10 \text{ days}}$, $\sigma = \frac{1}{3 \text{ days}}$. Here, λ^{-1} is the mean time period of contact to illness onset, which is estimated from Tian et al. (2020); γ^{-1} is the median recovery time of Remdesivir treatment, which is estimated from Beigel et al. (2020); the the mean latent period, σ^{-1}, is estimated from Lin et al. (2020). From data, we assume $\mu = 0.79$.

Result with varying cost after infection

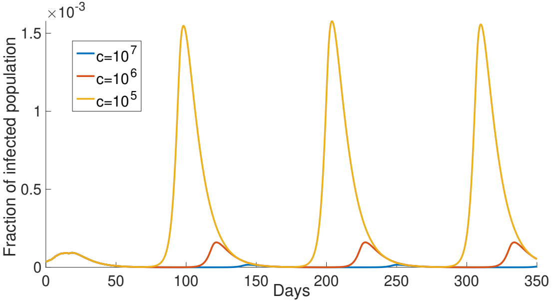

Figure: In this figure, we present the effect of varying the cost after infection. We can see that for smaller cost the infected population would be larger. And for larger cost the infected population will be smaller.

Result with varying time delay

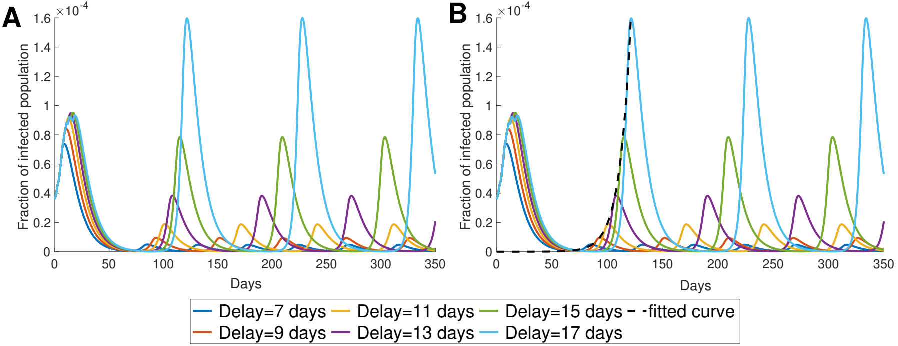

Figure: In this figure we show the impact of time delay. (A) We present the simulation with different delays. (B) We use exponential curve to fit the maximum in the first wave of the simulation. The fitted exponential curves are black dotted line.

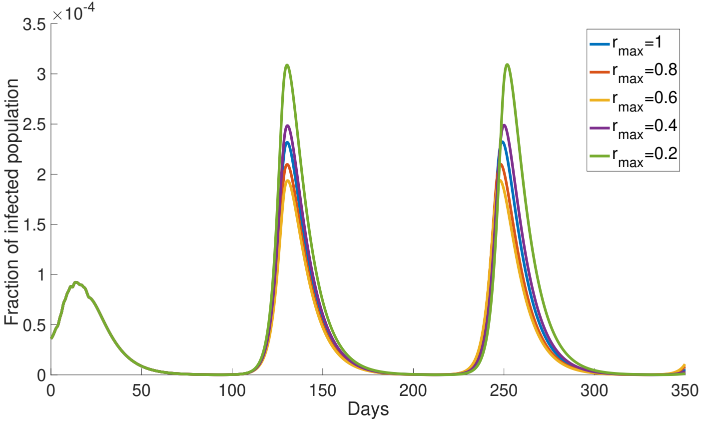

Figure: In this figure we present the simulation of the infected population with different r_{max} shown in the lenged. The infected population decreases as r_{max} decreases from 1 to 0.6. But the infected population increases as r_{max} decreases from 0.4 to 0.2.

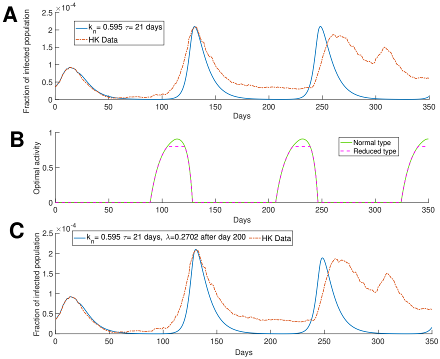

Figure: In this figure, we present the simulation with one group. (A) The comparison of the simulation and the data. (B) The optimal activity. (C) The comparison with report rate λ increase to 0.2702 after day 200.

- We can see that a new outbreak of infected population begins when the optimal activities of the normal type and reduced type are different.
- At this time, the normal type has more payoff than the reduced type. It is because when there is little infected population, the chance of being infected when having a higher z is less. Thus for a rational agent, he should increase his z to gain more payoff.
- Based on our model that the population can learn from the other population by comparing payoff, this also means that more and more people will like to have a higher z.
- As such, higher z leads to more infection, and a new outbreak occurs.

- When a new outbreak occurs, the number of the infected population is relatively high. The payoffs of the normal type and the reduced type become the same since it is not reasonable to have a higher z. And both populations will try to lower z in response to the increased risk of being infected.
- The infected population cannot sustain for a long period. Since under the rational population, the risk of transmission will be controlled by activity z.
- In summary, a new outbreak occurs when the number of the infected population is small and when the population increases their activity.

Thank You.

J. H. Beigel, K. M. Tomashek, L. E. Dodd, A. K. Mehta, B. S. Zingman, A. C. Kalil, E. Hohmann, H. Y. Chu, A. Luetkemeyer, S. Kline, et al., *Remdesivir for the treatment of COVID-19*, New England Journal of Medicine, 383 (2020), pp. 1813–1826.

G. Gaeta, *A simple SIR model with a large set of asymptomatic infectives*, arXiv preprint arXiv:2003.08720, (2020).

R. Li, S. Pei, B. Chen, Y. Song, T. Zhang, W. Yang, and J. Shaman, *Substantial undocumented infection facilitates the rapid dissemination of novel coronavirus (SARS-COV-2)*, Science, 368 (2020), pp. 489–493.

Q. Lin, S. Zhao, D. Gao, Y. Lou, S. Yang, S. S. Musa, M. H. Wang, Y. Cai, W. Wang, L. Yang, et al., *A conceptual model for the coronavirus disease 2019 (COVID-19) outbreak in Wuhan, China with individual reaction and governmental action*, International journal of infectious diseases, 93 (2020), pp. 211–216.

S. Tian, N. Hu, J. Lou, K. Chen, X. Kang, Z. Xiang, H. Chen, D. Wang, N. Liu, D. Liu, et al., *Characteristics of COVID-19 infection in Beijing*, Journal of Infection, (2020).